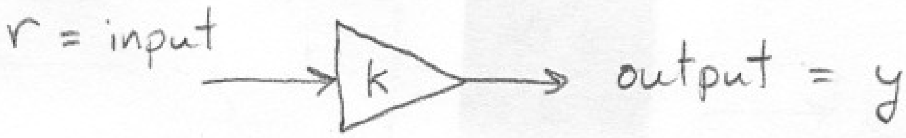
- Let's look at $G(s)$ in the frequency domain

$$G(s) = \frac{1}{s\tau + 1} = \frac{1/\tau}{s + 1/\tau} \Rightarrow G(j\omega) = \frac{1/\tau}{j\omega + 1/\tau}$$

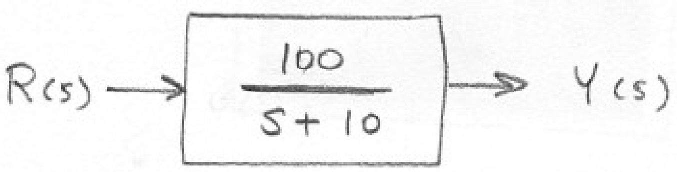
At $\omega = 0$ $\quad G(j\omega) = 1$

At $\omega = \infty$ $\quad G(j\omega) = \dfrac{1/\tau}{j\infty} = -\dfrac{j}{\infty} = -j0$

- The best way to think of a transfer function is an amplifier that has frequency dependence and complex gain. Let's first consider a linear amplifier



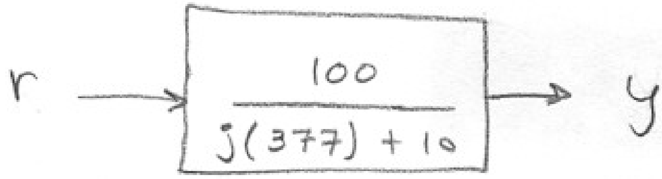
- If the gain is ten and the input is 5V, what is the output?
- If the gain is 20 dB and the input is 3V, what is the output?
- Now let's do the same for a transfer function



- If the input is a 5V DC voltage what is the output?

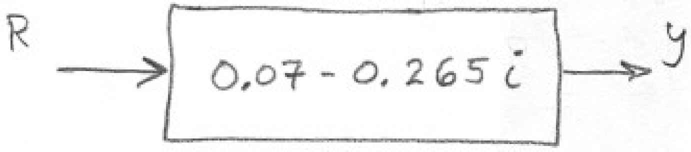
- If the input is 5V at 60 Hz what is the output? To solve this let's substitute in for $s = j\omega$

$$60\text{ Hz} \rightarrow \omega = 2\pi f = 2\pi(60\text{ Hz}) = 377\text{ r/s}$$



The number $\dfrac{100}{377j + 10} = \dfrac{100}{10 + 377j}\dfrac{(10 - 377j)}{(10 - 377j)} = 0.07 - 0.265i$

So we can replace the transfer function with a gain that is complex.



$$y = \underbrace{(5 + 0i)}_{\text{input}}\underbrace{(0.07 - 0.265i)}_{\text{gain}} = \underbrace{0.035 - 1.325i}_{\text{output}}$$

$$y = 1.326\text{ V} \measuredangle -88.5^\circ$$

Because the gain is complex the output is phase shifted wrt. the input.

- If the input voltage remains the same (5V), except the frequency is reduced to 10 Hz, how will that change the answer for y?

Plotting the transfer function is usually done by looking at the magnitude and phase. But typically

$20 \log_{10} \left| \frac{\text{output}}{\text{input}} \right|$ is plotted $\Rightarrow 20 \log_{10} |G(jw)|$

← magnitude

Note that $G(jw)$ is complex !!

The units of $20 \log_{10} |G(jw)|$ are referred to as decibels or dB. The magnitude is essentially constant before the corner frequency. The corner frequency occurs when the magnitude is 3dB below the low frequency magnitude. After the corner frequency the magnitude decreases at a rate of 20 dB/decade. Also the phase of the output wrt. the input is 0° for low frequencies, -45° at the corner frequency and -90° at high frequencies.

BODE plot
1st order
system